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Approximations by Lipschitz functions generated by extensions. (English. English summary)
Let \((X, d)\) and \((Y, d')\) be metric spaces, \(B \subset X\) and \(f: B \to Y\) a
Lipschitz function with Lipschitz constant \(\text{Lip}(f)\). One says that the
metric spaces \((X, d)\) and \((Y, d')\) have the Lipschitz extension property
(L.e.p.) if there exists a constant \(C = C(X, Y) > 0\) such that, for
every \(B \subset X\) and every Lipschitz function \(f: B \to Y\), there exists a
Lipschitz extension \(F: X \to Y\) such that \(\text{Lip}(F) \leq C \cdot \text{Lip}(f)\). The
author shows that if the pair of metric spaces \((X, d)\) and \((Y, d')\)
have the L.e.p. then for any bounded uniformly continuous function
\(f: X \to Y\) and any \(\varepsilon > 0\) there exists a Lipschitz function \(f: X \to Y\)
such that \(\sup_{x \in X} d'(f(x), F(x)) < \varepsilon\). The same result holds if \(X\)
is a locally convex space and \(Y = \mathbb{R}^n\). If \(X\) is a separable Fréchet
space and \(F: X \to \mathbb{R}\) is locally bounded and convex then the set of
differentiability points of the function \(F\) is dense in \(X\).

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