Kostake Teleman - a great Romanian geometer

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On December 24, 2007 our distinguished colleague and professor Kostake Teleman passed away.

It is our duty, in these moments, to remember his life, his permanent and enthusiastic work, in the field of Education and Mathematics, a career which spanned over half a century.

Kostake Teleman was born on May 19, 1933 in Bucharest. He graduated the primary school and ”Sfântul Sava” High School in Bucharest. As a pupil with an exceptional training he was a permanent correspondent and later a valuable fellow-worker at ”Gazeta Matematică și Fizică”.

Kostake Teleman graduated the University of Bucharest, the Faculty of Mathematics in 1955. Immediately after that he became an Assistant Lecturer at the Department of Geometry and Topology (Academician Gh. Vrânceanu was the Head of the Department at the time). He became a Lecturer and since 1967 he has been an Associate Professor. Professor K. Teleman taught, year after year, to generations of students systematic knowledge of Geometry, Differential geometry, Algebra, Algebraic geometry, General topology, Donaldson-Witten theory, Mechanics, Mathematical physics, Theory of relativity, Lie groups, Lie algebras, Foundations of geometry, Theory of curves, History of Mathematics.

In addition, his clear and deep perspective on the development of Differential geometry was remarkable. His honesty and moral behaviour were outstanding, too. During his work within the Department, all his students were permanently stimulated and helped in their studies and profession. As a teacher, prof. K. Teleman was a model of correctedness, highly esteemed by both his colleagues in the Department and his students. Having a remarkable mathematical training and excellent pedagogical abilities, prof. K. Teleman was a known and valuable personality in the Romanian mathematical education. His courses had a direct impact on the improvement of teaching in the field of geometry at our Faculty, in the last three decades of the last century.
Through his important activity and his brilliant mathematical talent, prof. K. Teleman ranks among our great geometers Gh. Tzitzeica, Gheorghe Vrănceanu, etc. His valuable monographs and lecture notes prove his deep mathematical culture. His clear and scientifically rigorous courses were very useful for students, master students, Ph.D. students and researchers in the field of Differential geometry and Topology.

Let us consider his monographs first:


In his monograph "Elements of topology and differentiable manifolds", K. Teleman discusses elements of General topology and the most important methods used in Algebraic topology and in the Geometry of differential manifolds. He also studies fibre bundles, simplicial polyhedra and cells complexes.

In the monograph "Methods and results in modern Differential geometry", K. Teleman looks into several important aspects in Differential geometry. Local and global methods are used to solve systems of equations with partial derivatives. Having many applications in the equivalence problem of geometrical structures and in getting a wide class of invariants, these methods played an important role in the development of modern differential geometry. By using the methods of Homological algebra and Topology, introduced in the theory of exterior differential systems, global results are likely to be obtained.

The book also contains applications in Mechanics and Relativity, based on notions of Differential geometry, useful for theories concerning the structure of the gravitational field and of the Universe.

Moreover, prof. K. Teleman discusses the theory of elliptic differential operators, which leads to developments in the field of Differential geometry and Topology.

K. Teleman was a student of Academician Gh. Vrănceanu and he continued the research initiated by his professor. As a student, he started to publish original papers on Differential geometry, since 1953. His areas of research in differential geometry were:

- the theory of spaces with affine connection;
- the symmetric Riemann spaces;
- the quaternionic projective spaces;
- the holonomy groups associated to an infinitesimal connection;
- the osculating fibre bundles to a Riemann surface;
- the motion group of Riemann spaces;
- the eccentric anomalies in the theory of relativity;
- the spaces with constant curvature;
- the Global differential geometry;
- the Algebraic topology;
- the Bäklund transforms;
- the classification of elementary particles;
- the conformal Lorentz geometry;
- the quantification of relativistic Lagrangeans.

In the field of algebraic topology he investigated the simultaneous generalization of homology and homotopy groups, the spectral sequences and the Casson handles.

One can form an idea on the breadth of his work by considering this simple enumeration of his research directions, in which he contributed greatly. Some of his works were joint contributions, such as the Geometry textbooks for the IX-th and the X-th classes of High School, written between 1979-1982.

His scientific work was published both in Romania and abroad; some of his papers appeared in important journals published abroad. His articles are still cited by many researchers, including several important geometers (Helgason, Greub, Halperin, Vanstone, Kobayashi, Nomizu etc).

A thorough description of prof. K. Teleman’s entire work is impossible to make. Subsequently, we can only concisely underline some of his major contributions, that we often quote and use.

1) Forty years ago, during a conversation between Academician Gh. Vrânceanu and Academician Gr. Moisil came under discussion a natural method, developed later by Academician Gh. Vrânceanu. This method gives the possibility to associate to every finite dimensional real algebra a space with constant affine connection. The structure constants of the algebra were considered as coefficients of the connection. Conversely, in a certain basis of a finite dimensional real algebra, the coefficients of a connection can be taken as structure constants. In this way, one can obtain a correspondence between certain algebraic and geometric properties. So, having a space with constant affine connection $A_n$, the property of being locally Euclidean is equivalent with the fact that the associated algebra is commutative and associative. Prof. K. Teleman devised a recurrent method, with respect to the dimension, meant to construct all commutative and associative finite dimensional algebras and hence all locally Euclidean spaces $A_n$ with constant connection.

His result proves the equivalence between the following problems:
i) Find the finite dimensional commutative and associative algebras;

ii) Find the abelian, linear groups which act locally transitive on complex affine spaces;

iii) Find the abelian, linear groups which act locally transitive on complex projective spaces;

iv) Find the local Euclidean spaces with constant connection;

v) Find the projective Euclidean spaces with affine connection, having the associated Thomas connection constant.

2) In a paper published in 1956, Gh. Vrăceanu generalised the results of Levi-Civita, concerning geodesically related Riemannian spaces. Gh. Vrăceanu proved that two non-trivial geodesically related Riemannian spaces, one of them being irreducible, have homothetic metrics. This result was improved by prof. K. Teleman in his paper "Ob adnoi teoreme Borelia-Lichnerowicza" (1958, Rev. Roum. de Math.). His result, cited by Italian, Russian and Japanese mathematicians proves that the curvature Riemann tensor, having a certain irreductibility property, determines the metric of the space, modulo a constant factor.

The condition of irreducibility due to Professor K. Teleman is considered with respect to a subgroup of the holonomy group generated by the curvature operators, without using operators provided by the covariant derivatives of the curvature. If one considers the results of Nomizu and Yano, who proved that the (1,3)-curvature tensor field, together with its covariant derivatives of any order, determine the Riemannian metric, up to a homothety, one will remark the importance of prof. Teleman’s approach.

3) The complex projective spaces $V_{2n} = P^n(C)$ and quaternionic spaces $V_{4n} = P^n(H)$, introduced by Gh. Vrăceanu and K. Teleman (Bull. Şt., 7 (1955)) can be endowed with symmetric Riemannian metrics, such that these spaces are considered as non-holonomic spaces of spheres $S^{2n+1}, S^{4n+3}$. This property is related to the fact that the spheres $S^{2n+1}, S^{4n+3}$ are the total spaces of some fibre bundles, with basis $P^n(C)$ and $P^n(H)$. These are the Hopf fibre bundles.

In the paper "On a class of symmetric Riemann spaces" (Revue Roum. de Math., 2 (1957)) prof. K. Teleman proved the following theorem:

Every closed symmetric Riemannian space $V_n$ can be defined as a non-holonomic subspace of a sphere, the metric of $V_n$ being equal with the metric of the sphere induced on the non-holonomic subspace.

This article is cited and used, among others, by Helgason, Kobayashi and Nomizu.

4) Grassmann manifolds $M = G_p(E^n)$ were studied by E. Cartan from a global Riemannian point of view. He proved that these spaces are symmetric.
Prof. K. Teleman extended Cartan’s results in a paper published in 1958, cited by Kobayashi and Nomizu, underlying the local properties of Grassmann manifolds $M$. A metric of $M$ is constructed, using two representations of $M$; the first of them is given in the space of automorphisms groups and this is non-holonomic. The second representation applies $M$ topologically and isometrically on the Euclidean space $E_n^2$.

5) In a paper published in 1963 (J. Math. Soc. Japan 15, 134-158), cited by Kobayashi and Nomizu, prof. K. Teleman continued an article from 1954 and classified the Riemannian spaces, with relatively many isometries. He used new and interesting techniques to study their isometry groups. He proved that if the isometry group $G$ of a Riemannian manifold $V$, with non constant curvature, has a linear irreducible subgroup as isotropy group of a point $x \in V$, then the dimension of $G$ is at most $n + p^2$, where $n = \dim V$ and $n = 2p$ or $n = 2p + 1$.

From a geometrical point of view, prof. K. Teleman studied the isometry group under which a Pfaff system is invariant. He proved that: a group of isometries of the sphere $S^{n-1}$, under which a non-trivial Pfaff system is invariant, and which is irreducible as a linear group, has the dimension at most $p^2$, where $n = 2p$ or $n = 2p + 1$.

6) In a paper published in Annali di Mat. (62 (1963), 379-421), cited and used among others by Greub, Halperin, Vanstone, Kobayashi, Nomizu, prof. K. Teleman studied the infinitesimal connections defined in differentiable fibre bundles of a given basis.

7) In 1973, the purpose of his research was to generalise and dualise a principle to get the invariants of a group, a principle introduced and used by Gh. Tzitzeica. Prof. K. Teleman obtained models with symmetry in the physics of elementary particles, using some of Gh. Tîţieca’s principles. Some of the models were the outcome of joint works with prof. M. Teleman.

We would like to point out that, due to his remarkable and seminal scientific and educational work, many students got interested in geometry and became valuable researchers. Some of his former students are teaching in our Faculty or in other Romanian universities; some of them work abroad (in the U.S.A., France, Italy, Switzerland, Great Britain, Australia etc).

It is difficult to estimate the number of scientists who, all over the world, have been initiated, permanently stimulated and helped by prof. K. Teleman, in their studies and professional activities, in different areas of research and teaching.

We attended his interesting lectures delivered in ”Gh. Vrânceanu” and ”Gh. Tzitzeica” Seminars of the Department of Geometry as well as his numerous communications at conferences organized by our University or other
Universities, with great pleasure. We noticed the importance he gave to the mathematical work of our great geometers Gh. Tzitzeica, Gh. Vrânceanu, J. Bolyai and others.

Prof. K. Teleman had always supported and encouraged scientific research. For his outstanding scientific and educational activity, we would like to express our gratitude, on behalf of generations of mathematicians, all of them former students. Prof. K. Teleman is considered one of our most important mathematicians, in the area of Geometry.

The members of the Department of Geometry at the University of Bucharest greatly appreciate the original and rich mathematical work of prof. K. Teleman, developed with equilibrium and harmony during more than a half of a century. Combining the classical and modern thought in both his fundamental research and educational activity, the work of prof. K. Teleman impresses by its constant vitality.

Apart from his impressive mathematical work and his distinguished educational activity, we are also delighted to emphasize his valuable moral qualities, his kindness, friendship, modesty and seriousness in tackling any problem.

His impressive scientific research, known and esteemed in our country and on an international level, his fruitful and valuable educational activity place him among our distinguished forerunners of the Romanian school of geometry: Gh. Tzitzeica, Gh. Vrânceanu, Al Myller, etc.

His departure is a great loss for the Romanian Mathematical field of research in general and for Geometry in particular. His bright figure will always be alive in our hearts.

Those close to him feel sad about his loss, but the most affected are his relatives, first of all his wife Mrs. Mihaela Teleman and his children Miss Ana Maria Teleman and Mr. Andrei Teleman, all three University professors.
List of publications of Professor Kostake Teleman

1. Papers


II. Monographs.


III. Didactic publications. (Lecture Notes, textbooks, articles for students)


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