Unusual damage characteristics of metallic materials with matrix displaying strength differential effects

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Dedicated to Professor Nicolaie D. Cristescu on his 85th birthday

Abstract - Tension-compression asymmetry associated to either twinning or deviations from Schmid law are observed in fully-dense metallic materials. In this paper, we numerically assess the influence of this tension-compression asymmetry of the plastic flow on void growth. Micromechanical finite-element analyses of three-dimensional unit cells are carried out. The plastic flow of the matrix is described by a criterion that accounts for tension-compression asymmetry induced by shear deformation mechanisms of the constituent grains. It is shown that if the matrix tensile strength is higher than its compressive strength, for axisymmetric loadings such that the third-invariant of the stress deviator, $J_3 > 0$, the rate of void growth is faster than in the case of axisymmetric loadings for which $J_3 < 0$. On the other hand, for porous polycrystals in which the matrix tensile strength is lower than its compressive strength, the opposite is true. All these unusual features of damage evolution are captured by Cazacu and Stewart [8] criterion for porous solids.

Key words and phrases : tension-compression asymmetry; porous materials; porosity evolution; ductility; damage.

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1. Introduction

Assessment of damage of polycrystalline metallic materials is one of the major concerns in material and structural engineering. Consequently, there have been intense efforts to gain fundamental understanding of damage mechanisms and develop models for prediction of damage evolution. Most of the studies concern modeling damage due to void growth in a von Mises material (e.g. Gurson [12, 13] and its various extensions such as Needleman and Tvergaard [20]). However, both numerical and experimental studies show that Gurson-type models underestimate the rate of void growth (e.g. Koplik and Needleman [17]). Using a rigorous limit-analysis approach, very recently Cazacu et al. [7] demonstrated that the plastic potential for a
porous solid with von Mises matrix containing spherical voids should involve a very specific coupling between the third-invariant of the stress deviator, $J_3$, and the mean stress, $\sigma_m$. Specifically, the analytical criterion developed predicts that for axisymmetric loadings, void growth is faster for loading histories corresponding to $J_3 > 0$ than for these corresponding to $J_3 < 0$. These trends were confirmed by finite-element (FE) unit cell calculations (for more details, see Alves et al. [1]).

Few studies have dealt with the description of the dilatational response of porous solids with Tresca matrix. In their seminal study, Rice and Tracey [27] conjectured that for very high triaxialities the rate of void growth in a Tresca material should be much faster than in a Mises material. Recently, Cazacu et al. [6] and Revil-Baudard and Cazacu [25] investigated the dilatational response of porous materials with Tresca and von Mises matrix for general 3-D conditions for both compressive and tensile states. Using rigorous scale bridging methods, it was demonstrated that the presence of voids induces dependence on all invariants, the noteworthy result being the key role played by the plastic flow of the matrix on all aspects of the mechanical response. Specifically, it was shown that if the matrix obeys the von Mises criterion, the shape of the cross-sections of the porous solid with the octahedral plane deviates slightly from a circle, and changes very little as the absolute value of the mean strain rate increases. However, if the matrix behavior is described by Tresca’s criterion, the shape of the cross-sections evolves from a regular hexagon to a smooth triangle with rounded corners. Furthermore, the role of $J_3$ on void evolution was explained (see Revil-Baudard and Cazacu [25]). FE unit cell calculations conducted for voided cubic cells obeying Tresca’s and Mises criterion were also compared with the predictions of the new models. Irrespective of the loading history, it was found that neglecting the local plastic heterogeneity leads to a drastic underestimation of the rate of void evolution (Cazacu et al. [4]).

Efforts have also been devoted to the development of yield criteria for porous materials with matrix which is plastically compressible. Examples include the criteria of Jeong and Pan [16], Barthelemy and Dormieux [3] (2004), Guo et al. [11] for porous media with matrix obeying Drucker-Prager’s yield criterion and associated flow rule. It is to be noted that in this case the tension-compression asymmetry of the matrix results from the yield criterion being sensitive to the mean stress. As such plastic deformation in the matrix is accompanied by volume changes (see also Cristescu [9]).

However, for certain fully-dense metallic polycrystals, a significant tension-compression asymmetry is observed although the plastic flow is incompressible. This strength-differential (S-D) effect observed at the macroscopic scale is due to the polarity of the plastic deformation mechanisms operating in the constituent grains, e.g. twinning (for pure titanium see Nixon et al. [21, 22]; Revil-Baudard et al. [26], etc.) or non-Schmid-type slip (see
Asaro and Rice [2]). Cazacu et al. [5] proposed an yield criterion that is pressure-insensitive, yet accounts for SD effects. The isotropic form of this criterion involves all principal values of the stress deviator and a parameter $k$, which is expressible only in terms of the ratio $\sigma_T/\sigma_C$ between the uniaxial yield in tension, $\sigma_T$, and that in compression $\sigma_C$ of the given material. In order to approximate the plastic response in the presence of randomly-distributed spherical voids inside such polycrystals, Cazacu and Stewart [8] carried out a limit analysis of a hollow sphere obeying the isotropic form of Cazacu et al. [5] yield criterion. In contrast to Gurson’s [13] criterion, Cazacu and Stewart’s [8] criterion involves all principal values of the stress deviator (or equivalently depends on both invariants of the stress deviator). Using Cazacu and Stewart [8] model, Revil-Baudard and Cazacu [24] numerically studied the role of the tension-compression asymmetry of the matrix on porosity evolution in a notched bar subjected to uniaxial tension. It was shown that even a slight difference between the flow stress in uniaxial tension and uniaxial compression (e.g. $\sigma_T/\sigma_C$ less than 0.7) leads to a distribution of damage that is strikingly different than that for a material with $\sigma_T/\sigma_C = 1$ (i.e. with von Mises matrix).

The aim of this paper is to investigate the combined influence of the tension-compression asymmetry of the matrix and loading path on the evolution of porosity. This is done using FE unit-cell calculations and Cazacu and Stewart [8] criterion for porous solids. The unit cell calculations are performed assuming that the plastic flow in the matrix is described bythe isotropic form of Cazacu et al. [5] yield criterion. In Section 2, we briefly recall the criterion for the matrix as well as Cazacu and Stewart [8] criterion. Discussion of the micromechanical unit-cell model, and the method of analysis follows. In Section 3 are presented simulations results for porous materials characterized by an SD ratio of the matrix ranging from 0.7 to 1.2. For each porous material, the macroscopic loadings imposed are such that the principal values of the macroscopic stresses $\Sigma_1$, $\Sigma_2$, $\Sigma_3$, follow a prescribed proportional loading history corresponding to a constant stress triaxiality $T_\Sigma = 1$. Specifically, the dilatational response is investigated under axisymmetric loadings ($\Sigma_1 = \Sigma_2$), where the axial overall stress, $\Sigma_3$, is adjusted so that a fixed ratio $\Sigma_3/\Sigma_1$ is maintained through the deformation process. To investigate the effects of the loading path, in particular the influence of the third-invariant, simulations are conducted for tensile loadings corresponding to the two possible orderings of the principal stresses i.e. ($\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 2.5$) and ($\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 0.25$), which correspond to either $J_3 > 0$ or $J_3 < 0$, respectively. It is shown that for the same imposed macroscopic stress loading, the distribution of the local plastic strain and local mean stress in the given porous material is strongly influenced by the tension-compression asymmetry ratio. Most importantly, it is revealed that there is a strong correlation between the SD parameter,
2. Problem Formulation and Method of Analysis

2.1. Cazacu and Stewart [8] criterion for porous materials

The main purpose of this study is to investigate the combined effects of the tension-compression asymmetry of the plastic flow of the incompressible matrix and loading path on porosity evolution for isotropic porous metallic materials containing randomly distributed spherical voids. Thereby, we consider that the plastically incompressible matrix is described by the isotropic form of Cazacu et al. [5] criterion, expressed as:

\[
m \sqrt{(|\sigma'_1| - k\sigma'_1)^2 + (|\sigma'_2| - k\sigma'_2)^2 + (|\sigma'_3| - k\sigma'_3)^2} = \sigma_T^2
\]

where \( k \) is a material parameter, \( \sigma'_1, \sigma'_2, \sigma'_3 \) are the principal values of the deviator of the Cauchy stress tensor, \( \sigma' = \sigma' - \sigma_m I \), with \( I \) denoting the second-order identity tensor. Using Eq. (2.1), it can be easily shown that the parameter \( k \) can be expressed as

\[
k = \frac{1 - h}{1 + h}, \quad \text{where} \quad h = \frac{\sqrt{2 - (\sigma_T/\sigma_C)^2}}{2(\sigma_T/\sigma_C)^2 - 1}
\]

(for more details, see Cazacu et al. [5]). The material constant \( m \) in Eq. (2.1) is defined as

\[
m = \sqrt{\frac{9}{2(3k^2 - 2k + 3)}}
\]

From Eq. (2.2) it follows that \(-1 \leq k \leq 1\) and that necessarily the range of variation of the tension-compression ratio should be: \(-1/\sqrt{2} \leq \sigma_T/\sigma_C \leq \sqrt{2}\).

In the case when the matrix material does not display SD effects i.e. \( \sigma_T = \sigma_C \), it follows that \( k = 0 \) and the isotropic form of Cazacu et al. [5] criterion reduces to the von Mises yield criterion. For materials with \( \sigma_T < \sigma_C \), the parameter \( k < 0 \); while for materials with \( \sigma_T > \sigma_C \), we have \( k > 0 \). For example, if the constituent grains of a FCC or BCC polycrystal with uniform texture deform by \{111\}{\langle 110 \rangle} slip obeying Schmid law, plastic flow has no tension-compression asymmetry, and the SD parameter \( k = 0 \) (von Mises behavior); if deformation twinning is a contributor to plastic deformation of
the constituent grains, the polycrystal displays strength differential effects and \( k \neq 0 \), which means that the plastic flow depends on the sign and ordering of all principal values of \( \sigma' \), or alternatively on both invariants of the stress deviator. Indeed, for a fully-dense isotropic FCC polycrystal deforming at single-crystal level only by \{111\}(11\bar{2}) twinning, Hosford and Allen [15] showed that the plastic flow is pressure-insensitive but the ratio \( \sigma_T/\sigma_C = 0.83 \), which corresponds to \( k = -0.3 \); on the other hand, in case of a fully-dense isotropic BCC polycrystal deforming only by \{111\}(11\bar{1}) twinning, \( \sigma_T/\sigma_C = 1.21 \) (the reciprocal of the ratio for the FCC polycrystal), which corresponds to \( k = +0.3 \).

To capture the combined effects associated to both tension-compression asymmetry due to specific plastic deformation mechanisms at single-crystal level and the tension-compression asymmetry due to the presence of voids, Cazacu and Stewart [8] developed an analytic yield criterion for porous solids containing randomly distributed spherical voids in a matrix that obeys the yield criterion (2.1) and associated flow rule. This criterion was derived using the kinematic non-linear homogenization approach of Hill [14] and Mandel [18], with the representative volume element being a hollow sphere.

Cazacu and Stewart [8] yield criterion for the porous material is of the form

\[
m^2 \sum_{i=1}^{3} \left( \frac{\sigma'_i}{\sigma_T} - k \sigma'_i \right)^2 + 2f \cosh \left( -z_s \frac{3P}{2\sigma_T} \right) - (1 + f^2) = 0, \tag{2.3}
\]

where \( f \) is the void volume fraction while \( z_s \) is a parameter, which depends on the sign of the pressure. Its expression is

\[
z_s = \begin{cases} 
1 & \text{if } P \geq 0 \\
\sqrt{\frac{3k^2 + 2k + 3}{3k^2 - 2k + 3}} & \text{if } P < 0
\end{cases}
\]

If the matrix material has no tension-compression asymmetry in its plastic response (\( \sigma_T = \sigma_C \)), then the SD coefficient \( k = 0 \), \( z_s = 1 \) and \( m = \sqrt{3}/2 \), so that Cazacu and Stewart [8] criterion reduces to the Gurson [13] criterion automatically. If the void volume fraction, \( f \), is equal to zero, Cazacu and Stewart [8] criterion reduces to the yield criterion of the matrix, given by Eq. (2.1).

2.2. Unit-cell model

Full three-dimensional (3D) FE cell model computations are conducted. It is assumed that the porous medium can be represented by a regular and periodic 3-D array of initially spherical voids in a fully dense solid matrix. Assuming that the inter-void spacing is the same in any direction, the unit
Figure 1: (a) Two-dimensional projection of the three-dimensional cubic unit cell model considered: $2C_0$ and $r_0$ denote the length of the undeformed cubic cell and the initial radius of the spherical void, respectively. (b) Finite-element mesh (13 699 nodes, 12 150 8 node hexahedral FE, selective reduced integration) of one-eighth of the unit cell with a spherical void at its center, with an initial void fraction of $f_0 = 0.01$.

cell is initially cubic with side lengths $2C_0$ and contains a single spherical void of radius $r_0$ at its center. Thus, the initial porosity is:

$$f_0 = \frac{\pi}{6} \left( \frac{r_0}{C_0} \right)^3$$

Cartesian tensor notations are used and the origin of the coordinate system is taken at the center of the void (see Fig. 1). Let $\mathbf{u} = \mathbf{x} - \mathbf{X}$ be the incremental displacement between the current ($\mathbf{x}$) and reference ($\mathbf{X}$) configurations, and $\mathbf{t}$ the Cauchy stress vector. Due to geometric and material symmetries, only one-eighth of the unit cell needs to be considered (Fig. 1), and thus the following symmetry conditions are prescribed on the planes $x = 0, y = 0, \text{and } z = 0$, respectively:

$$
\begin{align*}
    u_1(0, y, z) &= 0, & t_2(0, y, z) &= 0, & t_3(0, y, z) &= 0, \\
    u_2(x, 0, z) &= 0, & t_1(x, 0, z) &= 0, & t_3(x, 0, z) &= 0, \\
    u_3(x, y, 0) &= 0, & t_1(x, y, 0) &= 0, & t_2(x, y, 0) &= 0.
\end{align*}
$$

(2.5)

To simulate the constraints of the surrounding material, the faces of the unit cell, which are initially planes parallel to the coordinate planes, must remain parallel planes and shear free. Thus, the boundary conditions imposed on the three faces of the unit cell must be such that:

$$
\begin{align*}
    u_1(C_0, y, z) &= U_1^*(t), & t_2(C_0, y, z) &= t_3(C_0, y, z) = 0, \\
    u_2(x, C_0, z) &= U_2^*(t), & t_1(x, C_0, z) &= t_3(x, C_0, z) = 0, \\
    u_3(x, y, C_0) &= U_3^*(t), & t_1(x, y, C_0) &= t_2(x, y, C_0) = 0.
\end{align*}
$$

(2.6)
The time histories of the displacements $U_1^*(t)$, $U_2^*(t)$, and $U_3^*(t)$ in Eq. (2.6) are not prescribed but determined by the analysis in such a way that the macroscopic Cauchy stresses $\Sigma_1$, $\Sigma_2$, $\Sigma_3$ follow an imposed proportional loading history given by:

$$\frac{\Sigma_1}{\Sigma_3} = \frac{\Sigma_2}{\Sigma_3} = \rho \tag{2.7}$$

where $\rho$ is a prescribed constant, and the macroscopic true stress tensor $\Sigma$ is given by

$$\Sigma = \begin{bmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{bmatrix} \tag{2.8}$$

The macroscopic true stresses $\Sigma_1$, $\Sigma_2$, $\Sigma_3$ are calculated as:

$$\Sigma_1 = \frac{1}{C_2C_3} \int_0^{C_2} \int_0^{C_3} t_1 dz dy, \quad \Sigma_2 = \frac{1}{C_1C_3} \int_0^{C_1} \int_0^{C_3} t_2 dz dx,$$

$$\Sigma_3 = \frac{1}{C_1C_2} \int_0^{C_1} \int_0^{C_2} t_3 dx dy, \tag{2.9}$$

where $C_i = C_0 + U_i^*$ are the current cell dimensions. The void is considered to be traction-free. The porous material being isotropic, its mechanical response is fully characterized by the isotropic invariants of the overall stress $\Sigma$, i.e.:

$$\Sigma_m = \frac{1}{3} (\Sigma_1 + \Sigma_2 + \Sigma_3); \quad \Sigma_e = \sqrt{3J_2} = \sqrt{\frac{3}{2} (\Sigma_1'^2 + \Sigma_2'^2 + \Sigma_3'^2)}; \quad J_3 = \Sigma_1'\Sigma_2'\Sigma_3' \tag{2.10}$$

where $\Sigma_i' = \Sigma_i - \Sigma_m$, $i=1, 2, 3$. The following combinations of the above stress invariants will be used in the analysis, the stress triaxiality ratio $T_\Sigma$ defined as the ratio between the first and second stress invariants, i.e. $T_\Sigma = \Sigma_m/\sqrt{3J_2}$, the third-invariant of the stress deviator $J_3$ or Lode parameter: $\mu_\Sigma = \frac{3\sqrt{3}}{2} \frac{J_3}{(J_2)^{3/2}}$ (see Drucker [10]). The overall (macroscopic) principal strains and the macroscopic von Mises equivalent strain $E_e$ are calculated as:

$$E_1 = E_2 = \ln \left( \frac{C_1}{C_0} \right), \quad E_3 = \ln \left( \frac{C_2}{C_0} \right),$$

$$E_e = \frac{2}{3} |E_3 - E_1|. \tag{2.11}$$

The above-defined (Eq.2.5-2.9) macroscopic stress-based boundary conditions for the periodic cubic cell allow a rigorous definition of the boundary value problem to be solved by finite elements. The FE analyses were performed with DD3IMP (Menezes and Teodosiu [19], Oliveira et al. [23]), an in-house quasi-static elastic-plastic FE solver with a fully-implicit time integration scheme. In terms of the FE implementation, it is worth mentioning
that the degrees of freedom (dof) of all the nodes belonging to the same planar bounding surface of the cubic cell which contribute to the calculation of the prescribed macroscopic stresses (i.e. dofs associated with displacements $U^*_i$ in Eq. (2.6)) were summed in the global stiffness matrix, so the equations of all these dofs are replaced by only one equation/unknown variable. In this manner, it is ensured that all initially planar boundary surfaces remain strictly flat during the entire loading history. Additionally, for each time increment and for all equilibrium cycles, the three imposed macroscopic forces on each planar bounding surface of the cubic cell are continuously updated in order to ensure that on the final equilibrated configuration the macroscopic Cauchy stress ratios are constant. The macroscopic non-equilibrated forces are included in the fully-implicit Newton-Raphson algorithm in order to improve its convergence rate. Finally, a convergence criterion imposes that, for each planar surface, the ratio between the norm of the difference between the prescribed and effective macroscopic forces and the norm of the prescribed macroscopic force must be smaller than 0.001. Thus, it is strictly verified that the macroscopic stress triaxiality, $T_\Sigma$, remains constant throughout the deformation history.

As already mentioned, will analyze two different stress loading paths corresponding to the same stress triaxiality $T_\Sigma = 1$, i.e.: (a) $\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 2.5$, which corresponds to $\mu_\Sigma = 1$ ($J_3 > 0$), and (b) $\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 0.25$, which corresponds to $\mu_\Sigma = -1$ ($J_3 < 0$). Note that for loading (a) two principal values of the stress deviator $\Sigma'$ are compressive (negative), but the maximum principal value is tensile (positive) while for loading (b), two principal values of $\Sigma'$ are tensile (positive), but the minor principal value, which is compressive (negative), has the largest absolute value. The void volume fraction, $f$, is evaluated at the end of each time increment as:

$$f = 1 - \frac{V_{\text{matrix}}}{V_{\text{cell}}}$$

In the above equation, $V_{\text{cell}} = C_1C_2C_3$, while the volume of the deformed matrix, determined directly from the finite element formulation, is denoted $V_{\text{matrix}}$.

Using this FE cell model, the dilatational response of seven porous materials having the same initial porosity $f = 0.01$ (i.e. $r_0/C_0 = 0.271$), but characterized by different values of the SD parameter $k$ (see Table 1 for the corresponding tension-compression asymmetry ratios), is investigated. Note that in all the simulations and for the each loading path, only the material parameter $k$ is varied. All the other input material parameters are kept the same, i.e. the elastic properties (E= 200 GPa, $\nu = 0.33$, where E is the Young modulus and $\nu$ is the Poisson coefficient) and the material parameters involved in the isotropic hardening law describing the evolution of the matrix tensile yield strength with local equivalent plastic strain $\bar{\epsilon}^p$, i.e.
Parameter (Eq.(2.2)) \[ \sigma_T/\sigma_C \]
\[
| \begin{array}{c|c}
+0.9 & 1.4 \\
+0.6 & 1.36 \\
+0.3 & 1.2 \\
0 & 1. \\
-0.3 & 0.83 \\
-0.6 & 0.74 \\
-0.9 & 0.7 \\
\end{array} |
\]

Table 1: Values of the strength differential (or tension-compression asymmetry) parameter \( k \) investigated in this study, and the correspondent value of the tension-compression yielding ratio \( \sigma_T/\sigma_C \).

\[
Y = A (\epsilon_0 + \bar{\epsilon}^p)^n,
\]

(2.13)

where \( Y \) is the current matrix flow stress, and \( A, n \) and \( \epsilon_0 \) are material parameters. The numerical values of these parameters are: \( A = 881.53 \) MPa, \( n = 0.1, \epsilon_0 = 0.00037 \). Thus, all the differences in behavior between the seven porous materials are due solely to the specificities of the plastic flow of the matrix, which are described by the parameter \( k \).

Figure 1 shows the initial FE mesh of one-eighth of the unit cubic cell consisting of 12150 elements (8-node hexahedral finite elements; selective reduced integration technique, with 8 and 1 Gauss points for the deviatoric and volumetric parts of the velocity field gradient, respectively) and a total of 13699 nodes.

3. Results

3.1. Analysis of the dilatational response for axisymmetric tensile loading corresponding to \( T_\Sigma = 1 \) and \( J_3 > 0 \) (\( \Sigma_1 = \Sigma_2 \) and \( \Sigma_3/\Sigma_1 = 2.5 \))

First, we examine the dilatational response for a macroscopic tensile loading corresponding to the following constant ratios between the macroscopic principal stresses: \( \Sigma_1 = \Sigma_2 \) and \( \Sigma_3/\Sigma_1 = 2.5 \). This loading corresponds to a constant macroscopic stress triaxiality \( T_\Sigma = 1 \) and \( J_3 > 0 \) during the entire loading history. Note that such triaxiality is observed in tensile loading of blunt notched specimens (see also Needleman and Tvergaard [20]). The results obtained using the unit-cell model described in Section 2.2. are presented both in terms of the overall macroscopic response and local state fields.

Fig. 2 shows a comparison between the macroscopic effective stress vs. macroscopic effective strain curves (\( \Sigma_e \) vs. \( E_e \)) for the seven porous ma-
terials; Fig. 3 shows the evolution of the void volume fraction, \( f \), while Fig. 4 depicts the rate of void growth (\( \dot{f} \)) as a function of the macroscopic effective strain \( E_e \). For each material, the onset of failure is marked by a black dot while an open circle is indicative of final collapse. The procedure for the identification of these points, i.e. the onset of failure and final collapse, is based on monitoring the evolution with \( E_e \) of the areas of the cell’s cross sections that sustain the largest load, i.e. of the area of the effective cross section (cell’s cross section minus the void) and of the total cell’s cross section, respectively. The onset of failure and final collapse are defined as the equivalent strain at which there is a change in the slope on the evolution curves of the above defined cross-sections. This user-independent and consistent definition of the strains at the onset of failure and final collapse provides an objective way to compare different porous materials, and most importantly assess the influence of the loading path on failure.

First, let note that the results presented in Figs. 2 to Fig. 4 indicate a very strong coupling between porosity evolution and ductility and the tension-compression asymmetry ratio described by the parameter \( k \). It is clearly shown that the sign and the value of this parameter strongly affect the maximum ductility, damage accumulation and, consequently, the maximum strain sustained by each material. The ductility increases with increasing \( k \). In particular, for the limiting cases (i.e. highest SD contrast), the FE
Figure 3: Evolution of the void volume fraction with the macroscopic equivalent strain $E_e$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_\Sigma = 1$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k = +0.9$, +0.6, +0.3, 0, -0.3, -0.6, -0.9, respectively. The black and white dots mark the onset of failure and final collapse, respectively.

Figure 4: Evolution of the void growth rate ($\dot{f}$) with the macroscopic equivalent strain $E_e$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_\Sigma = 1$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k = +0.9$, +0.6, +0.3, 0, -0.3, -0.6, -0.9, respectively. The black and white dots mark the onset of failure and final collapse, respectively.
simulations show that for $k = 0.9$ ($\sigma_T/\sigma_C = 1.41$), the onset of failure occurs for an equivalent strain of around 0.18, while the material characterized by $k = -0.9$ ($\sigma_T/\sigma_C = 0.7$) displays super-plastic behavior (the onset of failure occurs at an effective macroscopic strain $E_e = 2.8$). In contrast, for the von Mises material ($k = 0$), the onset of failure was detected at $E_e = 0.40$. Such huge differences in failure strains correlate with the evolution of the porosity $f$ (Fig. 3) and that of the rate of void growth with the macroscopic effective strain $E_e$ (Fig. 4) for all cases. In the materials with $k \geq 0$ (i.e. $\sigma_T/\sigma_C \geq 1$) from the very beginning of the loading there is void growth. On the other hand, for materials with $k < 0$ (i.e. $\sigma_T/\sigma_C < 1$) the rate of void growth is almost constant for most of the loading. Moreover, only for the material with $k = -0.3$, coalescence occurs for $E_e < 1$, the strain as coalescence for the material with $k = -0.6$ are almost twice as large. The similarities in the macroscopic response between the materials with $k \geq 0$ (i.e. $\sigma_T/\sigma_C \geq 1$) are due to the fact that the corresponding SD ratios are less than 4% (see Table 1); same conclusion can be drawn concerning materials with $k = 0.6$ and $k = 0.9$ respectively.

It is also to be noted that in the case of materials with matrix characterized by $\sigma_T/\sigma_C \geq 1$ damage accumulation is much more gradual than in materials with matrix having $\sigma_T/\sigma_C < 1$. For example, the stress drop is more abrupt and failure more catastrophic (extent of the critical zone between the onset of coalescence and final collapse very limited) in the case of the material with $k = -0.3$, as compared with the von Mises porous or the porous material with $k = +0.3$ (see Fig. 2-3).

To further understand the reasons for the drastic differences in porosity evolution between the porous materials studied, we further compare the local state fields corresponding to the same level of the macroscopic effective strain $E_e = 0.20$. Fig. 5 shows the isocontours of the local equivalent plastic strain $\bar{\epsilon}^p$. The local equivalent plastic strain $\bar{\epsilon}^p$ is that associated with the effective stress for Cazacu et al. [5] criterion given by Eq. (2.1), i.e. to $\bar{\sigma}_e = \sqrt{\frac{9}{2(3k^2-2k+3)}} \sum_{i=1}^{3} (|\sigma'_i| - k\sigma'_i)^2$ based on the work-equivalence principle.

Results are shown for one lateral cross section, the vertical axis coincides with the axial loading axis while the horizontal axis coincides with the transverse loading axis as shown schematically in Fig. 5. Because of the drastic difference in the level of local plastic strains that develop in the seven materials studied, and in order to be able to better distinguish the distribution of the plastic zones within the domain, the scales are slightly different between materials (i.e. different maxima but the same minimum is considered for all materials). Examination of the isocontours reveals that:

- For the materials with matrix characterized by $\sigma_T/\sigma_C \geq 1$ there exists a zone in the vicinity of the void and along the vertical axis of the cross-section where yielding did not occur (white areas). Note that this
Figure 5: Isocontours of the local equivalent plastic strain $\bar{\varepsilon}^p$ corresponding to the same level of macroscopic strain $E_e = 0.2$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_2 = 1$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k = +0.9$, +0.6, +0.3, 0, -0.3, -0.6, -0.9, respectively. The white regions correspond to elastic regions ($\bar{\varepsilon}^p < 0.01$).
elastic zone shifts downwards with decreasing $k$, and for the porous von Mises material the elastic zone being contiguous to the void.

- On the other hand, for the materials with matrix characterized by $\sigma_T/\sigma_C < 1$ the entire domain (cell) is plastified.

Moreover, analysis of the isocontours of the local plastic strain shows very drastic differences in terms of the heterogeneity of the plastic. The local heterogeneity decreases with decreasing $k$, being the lowest in the material with $k = -0.9$. A measure of the heterogeneity in plastic deformation within the domain is the ratio between the maximum local plastic strain in the entire domain, $\varepsilon_{p_{\text{max}}}$, and the average of the local plastic strain, $\langle \varepsilon^p \rangle$, defined as:

$$\langle \varepsilon^p \rangle = \frac{1}{V} \int_V \varepsilon^p dV. \quad (3.1)$$

The highest is the plastic heterogeneity ratio $\varepsilon_{p_{\text{max}}}^p/\langle \varepsilon^p \rangle$, the most heterogeneity there is. As an example, for the materials with $k = 0.9, k = 0$ and $k = -0.9$, these heterogeneity ratios are of, respectively, 5.9, 5.2 and 2.2.

The distribution of the local stresses is also strongly influenced by the SD ratio of the matrix. Isocontours of the normalized mean stress $\sigma_m/Y_0$ are shown in Fig. 6. Again, there are drastic differences between the materials, which explain the marked differences in terms of damage evolution. Note that for materials with matrix characterized by $\sigma_T/\sigma_C \geq 1$ the local mean stress is positive in the entire domain. However, for materials with matrix characterized by $\sigma_T/\sigma_C < 1$ zones of negative mean stress develop near the void (along the axial loading axis). Moreover, as $k$ decreases the zone of maximum mean stress shifts farther and farther away from the void (e.g. for the material with $k = -0.9$, the zone corresponding to the maximum mean stress is the farthest from the void, while the zone of minimum (negative) mean stress occupies all the region near the front of the void and upwards along the axial loading axis). While the heterogeneity in local plastic deformation decreases with $k$ decreasing, the opposite holds true in what concerns the gradients in local mean stress. This correlates well with the results presented in Fig. 2-4, in particular explains the important differences in porosity evolution between the porous materials. Finally, note that although the materials with $k = \{-0.9, -0.6\}$ have similar macroscopic behavior, there are microstructural differences revealed by both the isocontours of the local plastic strain and mean stress, respectively.

In conclusion, although all porous materials were subjected to the same macroscopic tensile loading history corresponding to a constant macroscopic stress triaxiality $T_\Sigma = 1$ and $J_3 > 0$ during the entire deformation process, the specificities of the plastic flow of the matrix dramatically affects the local state. All the results presented highlight the strong correlation between the value of the macroscopic SD parameter $k$ and the local plastic
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Table 1: Isocontours of the local fields of the normalized mean stress $\sigma_m/Y_0$, corresponding to the same level of macroscopic strain $E_e = 0.2$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_\Sigma = 1$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k = +0.9, +0.6, +0.3, 0, -0.3, -0.6, -0.9$, respectively.

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<tr>
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<tr>
<td>+0.9</td>
<td>+0.6</td>
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<tr>
<td>0</td>
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Figure 6: Isocontours of the local fields of the normalized mean stress $\sigma_m/Y_0$, corresponding to the same level of macroscopic strain $E_e = 0.2$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_\Sigma = 1$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k = +0.9, +0.6, +0.3, 0, -0.3, -0.6, -0.9$, respectively.
Figure 7: Comparison between the evolution of the void volume fraction $f$ with the macroscopic equivalent strain $E_e$, obtained by cell calculations and Cazacu and Stewart [8] criterion for axisymmetric tensile loading at fixed $T_\Sigma = 2$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k=0, -0.3,$ and $0.3$.

strain heterogeneity and local mean stress, which in turn lead to markedly different void evolution and ultimately ductility of the porous materials (see Fig. 2-4). While the case of $T_\Sigma = 1$ and $J_3 > 0$ was discussed in detail, the same conclusions can be drawn for all moderate to high positive stress triaxialities. Furthermore, all these unusual damage characteristics are captured by the macroscopic criterion of Cazacu and Stewart (2009). As an example in Fig. 7 is shown a comparison between the porosity evolution with the macroscopic effective strain according to the model and the FE cell calculations corresponding to $T_\Sigma = 2$ and $J_3 > 0$ for materials with matrix characterized by $k=-0.3, k=0$ (von Mises), and $k=+0.3$. Note that the criterion describes well the porosity evolution, in particular that the rate of void growth is highest in the material with $k=+0.3$. The effect of triaxiality on void evolution is also well described by the model. Irrespective of the SD ratio, the higher the triaxiality the faster is the rate of void growth (see Fig. 8).

3.2. Analysis of the porosity and ductility evolution of porous materials subject to macroscopic axisymmetric tensile loading corresponding to $T_\Sigma = 1$ and $J_3 < 0$ ($\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 0.25$)

We analyze axisymmetric loadings at fixed triaxiality $T_\Sigma = 1$ corresponding to $J_3 < 0$, which corresponds to the following ratios between the macroscopic
Figure 8: Comparison between the evolution of the void volume fraction $f$ with the macroscopic equivalent strain $E_e$, obtained by cell calculations and Cazacu and Stewart [8] criterion for axisymmetric tensile loading at fixed triaxialities $T_\Sigma = 1, 2, 3$ and $J_3 > 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to: (a) $k = +0.3$, and (b) $k = -0.3$, respectively.
principal stresses: $\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 0.25$, i.e. the axial stress is lower than the lateral ones. Note that this loading case differs from the previous one only by the sign of $J_3$. Our aim is to investigate how the SD ratio, in particular the sign of $k$ affects the manner in which damage accumulates. Moreover, to study the cross-effect of the sign of $k$ and that of $J_3$ on damage growth.

In Fig. 9 is shown a comparison between the macroscopic equivalent stress $\Sigma_e$ vs. macroscopic equivalent strain $E_e$ curves, obtained using the cell model for all seven porous materials i.e. for $k=\{+0.9, +0.6, +0.3, 0, -0.3, -0.6, -0.9\}$. The evolution of the void volume fraction and the rate of void growth as a function of the macroscopic effective strain $E_e$, respectively, are shown in Fig. 10-11.

Note how strongly the specificities of the plastic flow of the matrix described by the parameter $k$ affect every aspect of the mechanical response of the given porous solid. Indeed, for this macroscopic stress path history, i.e. $T_\Sigma = 1$ and $J_3 < 0$, the fastest void growth rate and lowest ductility is observed for the porous material characterized by $k = -0.9$, which is exactly the opposite of what was found for the macroscopic loadings corresponding to $T_\Sigma = 1$ and $J_3 > 0$. On the other hand, for the material...
Figure 10: Evolution of the void volume fraction with the macroscopic equivalent strain $E_e$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_3 = 1$ and $J_3 < 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k= +0.9$, +0.6, +0.3, 0, -0.3, -0.6, -0.9, respectively. The black and white dots mark the onset of failure and final collapse, respectively.

Figure 11: Evolution of the void growth rate ($\dot{f}$) with the macroscopic equivalent strain $E_e$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_3 = 1$ and $J_3 < 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k= +0.9$, +0.6, +0.3, 0, -0.3, -0.6, -0.9, respectively. The black and white dots mark the onset of failure and final collapse, respectively.
Figure 12: Isocontours of the local equivalent plastic strain $\bar{\varepsilon}^p$ corresponding to the same level of macroscopic strain $E_e = 0.2$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_\Sigma = 1$ and $J_3 < 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k= +0.9, +0.6, +0.3, 0, -0.3, -0.6, -0.9$, respectively.

with $k = +0.9$ there is almost no damage evolution (see Fig. 10-11). Note that the porous materials with matrix such $\sigma_T/\sigma_C < 1$ show lower ductility than the von Mises material ($\sigma_T/\sigma_C = 1$), while all materials with matrix having $\sigma_T/\sigma_C > 1$ show higher ductility and much larger strains at both the onset of failure and final collapse. As an example, the porous material with $k = 0.3$, the onset of failure and the final collapse is detected at $E_e = 0.77$ and $E_\sigma = 1.0$, while for the material with $k = -0.3$, the onset of failure and the final collapse is detected at $E_e = 0.33$ and $E_\sigma = 0.41$, i.e. less than half the strain level observed in the latter material. Moreover, for the materials characterized by $k = 0.6$ and $k = 0.9$, coalescence or onset of failure did not occur although the calculations were conducted up to extremely large values of $E_e$. The material with $k = 0.9$ shows the lowest damage accumulation (void fraction converges asymptotically to the value $f/f_0 = 1.4$), the lowest rate of void growth and as a consequence enhanced ductility. All these results are consistent with the void volume fraction evolution (see Fig. 10) and the rate of void growth (Fig. 11) for the respective materials.
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Figure 13: Isocontours of the local fields of the normalized mean stress $\sigma_m/Y_0$, corresponding to the same level of macroscopic strain $E_e = 0.2$, obtained by cell calculations for axisymmetric tensile loading at fixed triaxiality $T_2 = 1$ and $J_3 < 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to $k = +0.9, +0.6, +0.3, 0, -0.3, -0.6, -0.9$, respectively.

To gain understanding of the large differences in porosity evolution between these porous materials for this loading case we also analyze the distribution of the local equivalent plastic strain $\bar{\varepsilon}^p$ corresponding to the same level of macroscopic effective strain $E_e = 0.2$ (Fig. 12), and the distribution of the local mean stress $\sigma_m/Y_0$ (Fig. 13) for all porous materials.

Note that the material where there is the least plastic heterogeneity is the material with $k = 0.9$ for which there is very little damage accumulation (see Fig. 10). For this material, the maximum and minimum values of the local equivalent plastic strain differ very little as well (0.20 and 0.13, respectively), the local normalized mean stress is much more homogeneous than in the other materials. This is the complete opposite of the case of the material with $k = -0.9$, for which the local plastic heterogeneity is much higher (maximum and minimum values of the local equivalent plastic strain of 2.06 and 0.03), the gradients of the normalized mean stress being
also much higher along the axial loading axis and much lower along the transverse loading axis. While in the case of the material with $k = 0.9$ the zone of largest mean stress is located away from the void, in the case of the material with $k = -0.9$ the maximum value of the mean stress is about twice as high and the zone of maximum damage is located right above the void along the axial axis. This explains the reason for rapid damage accumulation and much earlier fracture (final collapse) in the latter material.

Furthermore, it can be concluded that the heterogeneity in local plastic deformation increases with $k$ decreasing. Specifically, the local plastic strain heterogeneity is systematically higher in the porous materials with matrix characterized by $\sigma_T/\sigma_C < 1$ while in porous materials with $\sigma_T/\sigma_C > 1$ the local plastic strain is much more homogeneous.

Comparison between the predictions of the evolution of porosity according to Cazacu and Stewart [8] criterion and the FE cell calculations corresponding to axisymmetric loadings such that $J_3 < 0$ and fixed triaxialities $T_\Sigma = 1, 2, 3$, respectively are presented in Fig. 14. Note the very good agreement between the homogenized model and FE unit cell calculations.

4. Conclusions

In this paper, the effect of the tension-compression asymmetry of the plastic flow of the incompressible matrix on porosity evolution and the overall ductility of porous materials was investigated. First, unit cell model calculations were performed. The plastic flow of the incompressible matrix was considered to obey the isotropic form of Cazacu et al. (2006) (see [5]) yield criterion, which accounts for tension-compression asymmetry through a parameter $k$, which is intimately related to specific single-crystal plastic deformation mechanisms. The imposed macroscopic loadings were such that the principal values of the macroscopic stresses $\Sigma_1$, $\Sigma_2$, $\Sigma_3$ followed a prescribed proportional loading history. Detailed analyses were presented for tensile loadings corresponding to ($\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 2.5$) and ($\Sigma_1 = \Sigma_2$ and $\Sigma_3/\Sigma_1 = 0.25$), which correspond to the same value of the stress triaxiality, $T_\Sigma = 1$ and loadings at either $J_3 > 0$ or $J_3 < 0$ (Lode parameter $\mu_\Sigma = +1$ and $\mu_\Sigma = -1$, respectively). It was clearly shown that irrespective of the imposed macroscopic loading, the tension-compression asymmetry in the plastic flow of the matrix, described by the parameter $k$, has a very strong influence on all aspects of the mechanical response of the porous solids.

Furthermore, a very strong effect of the loading path, in particular of the third-invariant $J_3$ on porosity evolution and ultimately the material’s ductility was observed. Specifically, for materials with matrix such that $\sigma_T/\sigma_C < 1$ ($k < 0$) for axisymmetric loadings at $J_3 > 0$, damage growth is much slower than in the case of loadings at $J_3 < 0$, which in turns af-
Figure 14: Comparison between the evolution of the void volume fraction $f$ with the macroscopic equivalent strain $E_e$, obtained by cell calculations and Cazacu and Stewart [8] criterion for axisymmetric tensile loading at fixed triaxialities $T_\Sigma = 1, 2, 3$ and $J_3 < 0$ for materials characterized by different tension-compression asymmetry ratios corresponding to: (a) $k=+0.3$, and (b) $k = -0.3$, respectively.
fects the overall ductility. The reverse holds true for materials with matrix characterized by $\sigma_T/\sigma_C > 1$ ($k > 0$) or von Mises material ($\sigma_T/\sigma_C = 1$, so $k = 0$).

All those trends were captured by the analytical criterion of Cazacu and Stewart [8] for porous materials with matrix displaying tension-compression asymmetry.

Moreover, the simulations results provide insights onto material design such as to control "damage" under given macroscopic loading conditions. While in the examples shown here the source of tension-compression asymmetry of the matrix was due to twinning at single-crystal level, this strength-differential in the matrix may arise from other single-crystal plasticity mechanisms, e.g. when different components of the applied stress affect the single crystal plastic deformation by climb and glide.

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References


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