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To Professor Cassius Ionescu Tulcea on the Occasion of His 90$^{\text{th}}$ Birthday

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Cassius Tocqueville Ionescu Tulcea − a legendary name of mathematics. This article is dedicated, with love and admiration, to this great mathematician, on the occasion of his 90$^{\text{th}}$ birthday.

1. Let us begin by emphasising the great personality of Cassius T. Ionescu Tulcea (CTIT in the sequel) as a teacher. Those who were fortunate enough to have been his students remember that they were absolutely mesmerised by his impeccable and noble appearance, the scientific accuracy of his lectures and the outstanding delivery in front of the audience.

2. Now, let us highlight the most important aspects of the biography of CTIT. He was born on October 14, 1923, in Bucharest, in the family of a lawyer. He graduated from the Sf. Sava high school in 1942 and, in that most difficult period for Romania, he studied Mathematics at the Faculty of Sciences (Section of Mathematics) of the University of Bucharest, between 1942 − 1946. Immediately after graduation, he was appointed assistant professor at the aforementioned University, namely at the Calculus of Probability Department (head of the department was Octav Onicescu) between 1946 −1950. Then he became lecturer at the same department between 1950 − 1951. Afterwards he moved to the Analysis Department (head of the department was Miron Nicolescu) where he worked as an associate professor between 1952 − 1957 until he left the country. Additionally, CTIT worked as a researcher at the Institute of Mathematics of the Romanian Academy, between 1949 − 1957.

In 1957, the Romanian adventure of CTIT ended and his American adventure began. The famous American mathematician of Swedish origin Einar Hille, ex president of the American Mathematical Society, attended the Congress of the Romanian Mathematicians in Bucharest, 1956. On this occasion, Einar Hille, who knew the work of CTIT and appreciated it enormously, invited CTIT in the United States to participate in a special program on Functional Analysis at the famous Yale University, together with a team of distinguished mathematicians from different parts of the world. As a consequence, in 1957, CTIT moved to USA together with his

wife Alexandra Ionescu Tulcea (born Alexandra Bagdasar) who has been his student.

During this period he worked as a research associate and visiting lecturer between 1957 − 1961 at the Yale University. CTIT obtained his Ph. D. degree in 1959 with the thesis Semigroups of Operators under the guidance of Einar Hille (one of the pioneers of the semigroups of operators theory). As a doctoral student of Einar Hille, CTIT is in an illustrious company: H. F. Bohnenblust, I. Segal and other important mathematicians obtained their Math. Ph. D. title under the guidance of Einar Hille. Obviously, it was a pure formality for CTIT to get his Ph. D. degree, due to his previous work.

One year after that, in 1960, Robert Langlands and George Maltese got their Ph. D. degree under the guidance of CTIT. During 1961 − 1964, CTIT was an associate professor at Pennsylvania University, Philadelphia, a top American university. Here, at various periods of time the, Mathematics Department had counted among its faculty, J.A. Clarkson and A. Zygmund, H.A. Rademacher and I.J. Schoenberg, later E. Calabi and R.K. Kadison. After 1964, CTIT moved to the University of Illinois at Urbana − Champaign and then to Northwestern University, Evanston.

3. Here is a list of the books written by CTIT:

- *Hilbert Spaces* (1956, in Romanian);
- *The Calculus of Probability and Applications* (1956, jointly with Octav Onicescu and Gheorghe Mihoc, in Romanian);
- *Calculus* (1968, jointly with Robert Bartle);
- *An Introduction to Calculus* (1969, jointly with Robert Bartle);
- *Topics in the Theory of Lifting* (1969, jointly with Alexandra Ionescu Tulcea);
- *Honors Calculus* (1970, jointly with Robert Bartle);
- *Sets* (1970, jointly with William Fairchild);
- *Topology* (1971, jointly with William Fairchild);
- *A Book on Casino Gambling Written by a Mathematician and a Computer Expert* (five editions 1976−1980, jointly with Virginia L. Graham);
- *A Book on Casino Craps, Other Dice Games and Gambling* (1981);
- *A Book on Casino Blackjack* (1982).

The two monographs written in Romanian are of an exceptional value. The first one (dedicated to the theory of Hilbert Spaces) continues to be of great interest due to its accuracy and contemporary significance. The second one contains a great variety of topics. The monograph dedicated to the Theory of Lifting, written jointly with Alexandra Ionescu Tulcea is quoted in numerous papers of mathematical analysis.

4. Concerning the doctoral students of CTIT:

A. Although he did not have a Ph. D. degree during his stay in Romania, CTIT was the main advisor of a few mathematicians who obtained their Ph. D. degree at the University of Bucharest, under joint guidance:

- Nicolae Dinculeanu (1957) under the joint guidance of Octav Onicescu and CTIT;
- George Ciucu (1957) under the joint guidance of Octav Onicescu and CTIT;
- Radu Theodorescu (1958) under the joint guidance of Octav Onicescu, Gheorghe Mihoc and CTIT.

B. The most prominent mathematician who got his Ph. D. degree under the guidance of CTIT, during his stay in the USA, is the already mentioned Robert Langlands (with his famous program). We mention too George Maltese (CTIT-first advisor and Einar Hille-second advisor) and William Fairchild.

5. In the sequel, we shall try to outline briefly the research work of CTIT, who is an expert in Functional Analysis, Measure and Integration Theory and Probability Theory, all the three fields being deeply interrelated.

In the field of *Functional Analysis* we begin with the highlighting of the aforementioned monograph on the Hilbert Spaces. This book can be regarded as a research work at the time of its publication, being a fundamental work for the modern learning of Functional Analysis and a starting point for many researchers in the field of Operator Theory. CTIT contributed to the Theory of Spectral Operators of Nelson Dunford (extended to locally convex spaces), as well as to the field of Operator Algebras. The noteworthy contribution of CTIT to the theory of Operator Semigroups captured the interest of Einar Hille, one of the founding fathers of this theory (see also the doctoral thesis of CTIT). We mention too the contributions of CTIT to the theory of representation of linear and continuous operators on Lebesgue Spaces of vector valued functions, which he generalized for spaces of Vector Fields. Finally, we mention the contribution to the theory functions of Positive Type.

In the field of *Measure and Integration Theory*, CTIT worked together with Alexandra Ionescu Tulcea on the subject of Lifting Theory, obtaining

fundamental results published in several articles and in their monograph dedicated to this subject. CTIT worked also on the theory of Abstract Integration, introducing a topological semigroup-valued integral, which generalizes Rickarts, Kolmogorovs and Cotlars abstract integrals. A further contribution in this direction is a theory of an ordered vector space-valued integral which enabled CTIT to generalize the F. Riesz representation theorem and to obtain, as a byproduct, a spectral representation of elements in ordered vector spaces. CTIT published in Atti della Accademia Nazionale dei Lincei a paper concerning probabilities on product spaces. In this paper he proves a theorem which is a correction of an error made by Doob. The last chapter of Jacques Neveu's 1964 monograph concerning The Foundations of Calculus of Probability is dedicated to this theorem.

The role played by CTIT in the development of *"Probability Theory"* starts with the monograph written with Octav Onicescu and Gheorghe Mihoc (already mentioned). First of all, we must refer to the valuable contributions of CTIT to the theory of Chains with Complete Connections (put forward by Octav Onicescu and Gheorghe Mihoc). These contributions are relevant to the general theory as well as to the ergodic theory of these chains. Regarding this subject, CTIT published jointly with Gheorghe Marinescu, an important article in Annals of Mathematics in which the ergodicity of some classes of operators in connection with the chains with complete connections is studied. In the same spirit of intersection between Functional Analysis and Theory of Probability, CTIT obtained ergodic properties of representations of locally compact groups, generalizing some results of John von Neumann obtained for compact groups. It is worth mentioning that CTIT obtained some results concerning the Moment Theory. Last but not least, we mention the contributions of CTIT to Gambling Theory, obtained in the framework of the Probability Theory (see the three aforementioned books on this subject).

We have attempted to review concisely the biography and the work of a great personality. The Romanian mathematical community expresses its gratitude to professor Cassius Ionescu Tulcea for all his activity.

Many Happy Returns, Professor!

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