An invitation to Moisil's logic

Ioana Leuştean

Abstract

The first system of many-valued logic, a 3-valued propositional calculus, was defined by J. Łukasiewicz in 1920. The n-valued propositional logic was published by J. Łukasiewicz in 1929 and it was generalized to infinitely many values by J. Łukasiewicz and A. Tarski in 1930. Gr.C. Moisil was the first to consider the algebras connected with Łukasiewicz logic. In 1940, he defined 3-valued and 4-valued Łukasiewicz algebras, which he later generalized to the n-valued and infinitely valued case. In 1965, A. Rose proved that the algebras defined by Moisil were not adequate for the n-valued Łukasiewicz logic with $n \geq 5$. The algebras defined by Moisil, which are nowadays called *Łukasiewicz-Moisil algebras*, evolved as a distinct theory in algebraic logic.

A *Łukasiewicz-Moisil algebra of order* $n+1$ is a structure

$$(L, \vee, \wedge, ^*, \varphi_1, \ldots, \varphi_n, 0, 1)$$

where $(L, \vee, \wedge, ^*, 0, 1)$ is a De Morgan algebra and $\varphi_1, \ldots, \varphi_n$ are lattice endomorphisms defined on L, satisfying some additional axioms. Notably, the operations $\varphi_1, \ldots, \varphi_n$ take values in the Boolean center $C(L)$, which is the set of all elements satisfying the law of excluded middle.

One of the most powerful tools in the theory of Łukasiewicz-Moisil algebras is *the determination principle*: if L is a Łukasiewicz-Moisil algebra of order $n+1$ and x, y are elements of L then

$$x = y \Leftrightarrow \varphi_i(x) = \varphi_i(y) \text{ for any } i \in \{1, \ldots, n\}.$$

The *n-valued Moisil logic* is the logic that stands to the n-valued Łukasiewicz-Moisil algebras as the Boolean logic stands to Boolean algebras. The logical interpretation of Moisil's determination principle provides an alternative way to reason about vagueness in the context of many-valued logics: a many-valued event is uniquely characterized by a family of Boolean events, its *Boolean nuances*.

In our talk, we present Moisil's logic and it's algebra, surveying old and new results.

Ioana Leuştean
University of Bucharest